A Universal Approach to Developing Fast Algorithm for Simplified Order-16 ICT

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Outline

- Introduction
- Simplified order-16 ICT
- Proposed approach
- Complexity analysis
- Conclusion and future work

Introduction

- Integer Cosine Transform (ICT)
  - Pro: Integer arithmetic implementation
    Avoid mismatch between encoder and decoder
    Good energy compaction capability if well-designed
    Fast algorithms can be developed in the similar way for DCT
  - Con: Orthogonality depends on the elements of transform matrix, when the ICT is larger than order-4.

- ICT for video coding
  - Order-4 and Order-8 ICTs in H.264
  - Order-8 ICT in Audio Video Standard (AVS)
  - Order-16 ICT: efficient tool especially for HD video coding

- Simplified Order-16 ICT
  - Pro: Simpler while preserving the advantages of order-16 ICT
  - Con: Cannot develop fast algorithm in the similar way for DCT/ICT

Simplified Order-16 ICT

General transform matrix

- Contain at most 15 different integers
- Naturally orthogonal
- Typical elements: represented by 4~5 bits

o	o	o	o	o	o	o	o	o	o	...
a	b	c	d	e	f	g	h	-h	-g	...
i	j	k	l	-l	-k	-j	-i	-i	-j	...
e	f	g	h	-a	-b	-c	-d	d	c	...
m	n	-n	-m	-m	-n	n	m	m	n	...
c	d	-a	-b	-g	-h	e	f	-f	-e	...
j	-l	-i	-k	k	i	l	-j	-j	l	...
h	g	-f	-e	d	c	-b	-a	a	b	...
o	-o	-o	o	o	-o	-o	o	o	-o	...
g	-h	-e	f	c	-d	-a	b	-b	a	...
k	-i	l	j	-j	-l	i	-k	-k	i	...
b	-a	-d	c	-f	e	h	-g	g	-h	...
n	-m	m	-n	-n	m	-m	n	n	-m	...
d	-c	b	-a	-h	g	-f	e	-e	f	...
l	-k	j	-i	i	-j	k	-l	-l	k	...
f	-e	h	-g	b	-a	d	-c	c	-d	...

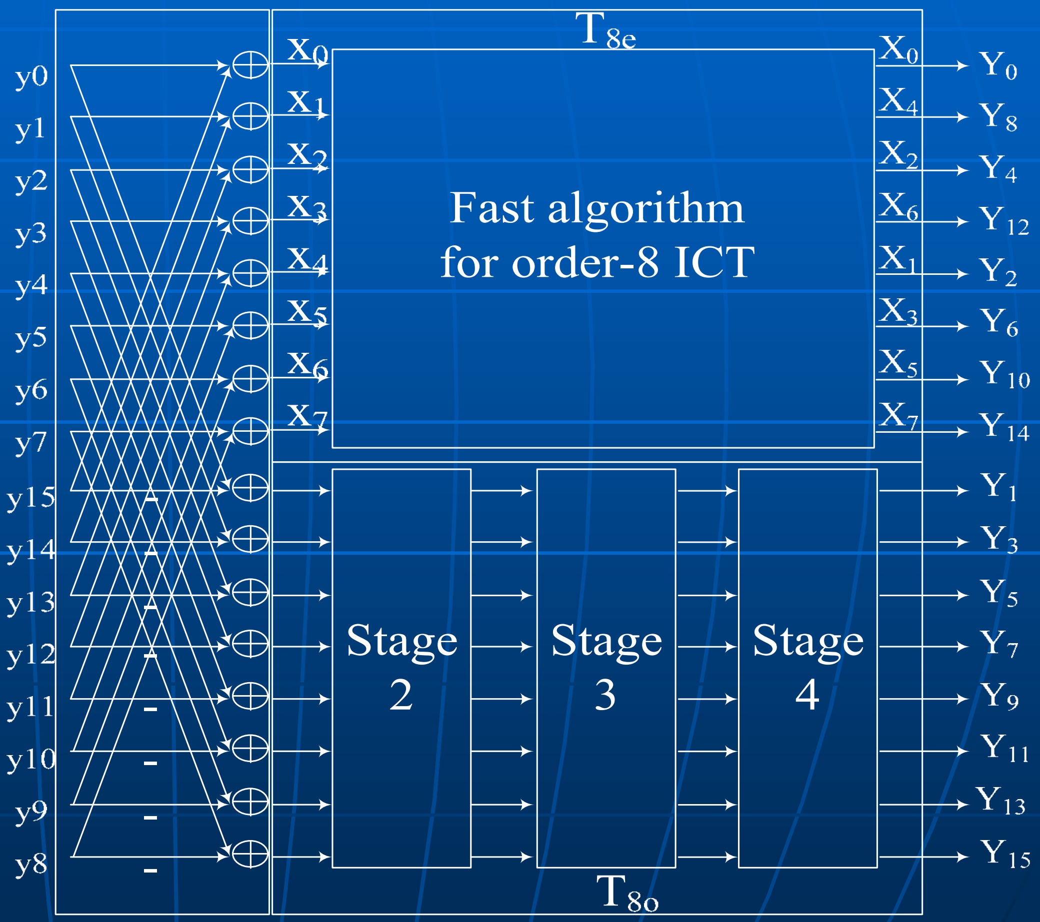
Flow Diagram

- Even part (T_{8e})

o	o	o	o	o	o	o	o
i	j	k	l	-l	-k	-j	-i
m	n	-n	-m	-m	-n	n	m
j	-l	-i	-k	k	i	l	-j
o	-o	-o	o	o	-o	-o	o
k	-i	l	j	-j	-l	i	-k
n	-m	m	-n	-n	m	-m	n
l	-k	j	-i	i	-j	k	-l

- Odd part (T_{8o})

a	b	c	d	e	f	g	h
e	f	g	h	-a	-b	-c	-d
c	d	-a	-b	-g	-h	e	f
h	g	-f	-e	d	c	-b	-a
g	-h	-e	f	c	-d	-a	b
b	-a	-d	c	-f	e	h	-g
d	-c	b	-a	-h	g	-f	e
f	-e	h	-g	b	-a	d	-c



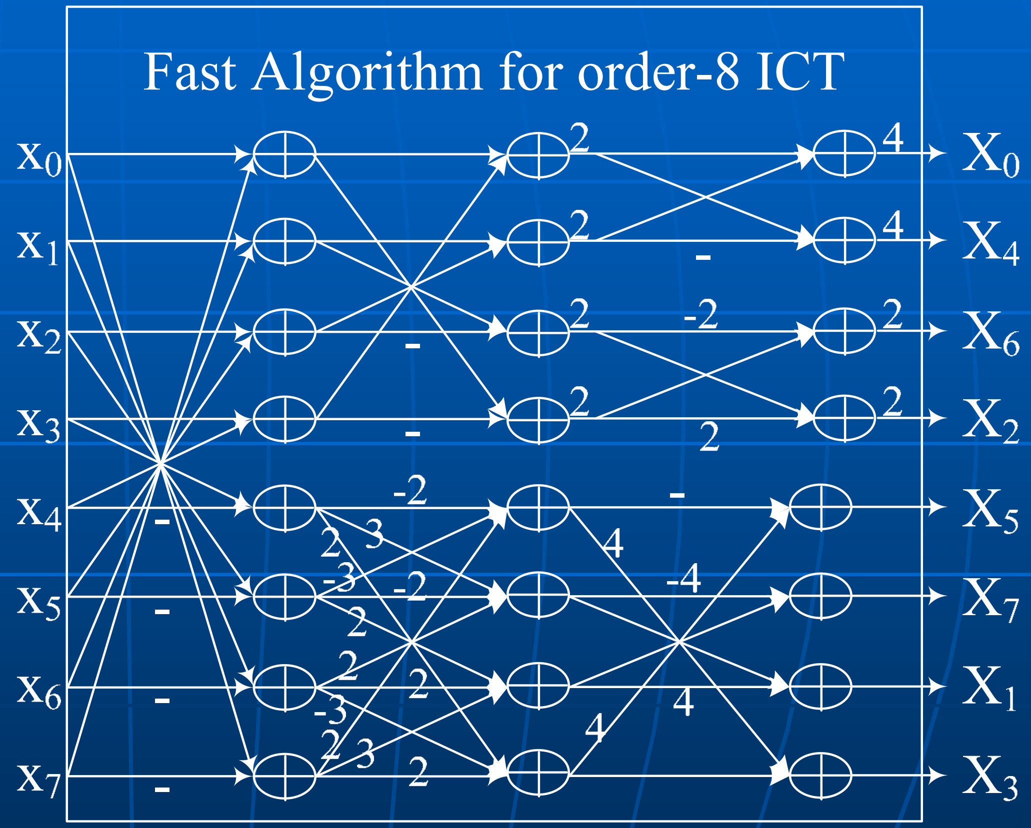
Steps of the Proposed Approach

- Separate the 2-D transform into 2 1-D transforms
- Using 8 butterflies (left block) to exploit the symmetries w.r.t the dash line in the general transform matrix
- Fast algorithm for the even part (upper-right block) that is exactly the general transform matrix of an order-8 ICT.
  - Borrow order-8 ICTs and their fast algorithms, e.g., order-8 ICTs in H.264 or AVS
  - Otherwise, the fast algorithm will be developed in the same way for the odd part.
- Fast algorithm for the odd part (bottom-right block)

An Order-8 ICT and its Fast Algorithm

- Order-8 ICT in H.264 is borrowed as the even part (upper-right block) of the simplified order-16 ICT

8	8	8	8	8	8	8	8
12	10	6	3	-3	-6	-10	-12
8	4	-4	-8	-8	-4	4	8
10	-3	-12	-6	6	12	3	-10
8	-8	-8	8	8	-8	-8	8
6	-12	3	10	-10	-3	12	-6
4	-8	8	-4	-4	8	-8	4
3	-6	10	-12	12	-10	6	-3



Fast Algorithm for the Odd Part

- T_{8o}: a type of dyadic transform having different structures with the odd part of DCT/ICT

- Decompose T_{8o} to the multiplication of three 8x8 matrices instead of butterfly operations

$$T_{8o}=M_2 x M_3 x M_4$$

  M_2, M_3, M_4: Stage 2~4 in the flow diagram

- Considerations for M_2, M_3, M_4
  - Contain integers only
  - Small magnitude to avoid multiplications
  - Be sparse

Fast Algorithm for the Odd Part (Cont.d)

- Constraints for M_2, M_3, M_4: each has the same properties of T_{8o}
  - Orthogonality
  - Basis vectors has the same length (length of vector *a*: $a \times a^T$)
- Conditions (necessary, not sufficient) of existence under the constraints (n_{8o}, n_2, n_3, and n_4 represent the length of each matrix)

$$|\det(T_{8o})| = |\det(M_2)| \times |\det(M_3)| \times |\det(M_4)| \quad (1)$$

$$\Rightarrow \quad n_{8o}^4 = n_2^4 \times n_3^4 \times n_4^4 \quad (2)$$

$$\Rightarrow \quad n_{8o} = n_2 \times n_3 \times n_4 \quad (3)$$

n_{8o} is the product of at least 3 prime numbers

- (3) means n_2, n_3, and n_4 are much smaller than n_{8o}, which indicates the elements in the three matrices have very small magnitudes and may also contain many zeros.

Fast Algorithm for the Odd Part (Cont.d)

- Search M_2, M_3, M_4, start from M_4

  $T_{8o} x (M_4)^{-1} = M_2 x M_3 x M_4 x (M_4)^{-1} \rightarrow T_{8o} x (M_4)^T / n_4 = M_2 x M_3$ (4)

  - Notice, in (4)
    - $T_{8o} x (M_4)^T / n_4$ contains only integers
    - $(M_4)^T$ can be regarded as a set of column vectors
  - Search M_4
    - Establish a set of column vectors $\{b_i\}$, satisfying
      - Length of b_i is n_4, i.e., $b_i x b_i^T = n_4$
      - $T_{8o} x b_i / n_4$ contains only integers
    - Pick out eight orthogonal column vectors from $\{b_i\}$ to form $(M_4)^T$ and thus get M_4.
- Search M_3 and M_2 (similar to searching M_4)

$[T_{8o} x (M_4)^T / n_4] x (M_3)^{-1} = M_2 x M_3 x (M_3)^{-1} \rightarrow [T_{8o} x (M_4)^T / n_4] x (M_3)^T / n_3 = M_2$ (5)

An example

- Even part (T_{8e})

8	8	8	8	8	8	8	8
10	9	6	2	-2	-6	-9	-10
10	4	-4	-10	-10	-4	4	10
9	-2	-10	-6	6	10	2	-9
8	-8	-8	8	8	-8	-8	8
6	-10	2	9	-9	-2	10	-6
4	-10	10	-4	-4	10	-10	4
2	-6	9	-10	10	-9	6	-2

- Odd part (T_{8o})

11	11	11	9	8	6	4	1
8	6	4	1	-11	-11	-11	-9
11	9	-11	-11	-4	-1	8	6
1	4	-6	-8	9	11	-11	-11
4	-1	-8	6	11	-9	-11	11
11	-11	-9	11	-6	8	1	-4
9	-11	11	-11	-1	4	-6	8
6	-8	1	-4	11	-11	9	-11

- $\{a, b, c, d, \ldots, n, o\} =$ {11, 11, 11, 9, 8, 6, 4, 1, 10, 9, 6, 2, 10, 4, 8}
- Even part (T_{8e})

  order-8 ICT in AVS
- Odd part (T_{8o})

  $|(T_{8o})| = 9.9049 \times 10^{10}$

  $n_{8o} = 561 = 3 \times 11 \times 17$

  $n_2 = 17$, $n_3 = 3$, $n_4 = 11$

  Search M_2, M_3, M_4 using the method in the previous page

An Example (Cont.d)

$$T_{8o} = M_2 x M_3 x M_4 =$$

$$\begin{bmatrix} -2 & 0 & 1 & -1 & -1 & 3 & -1 & 0 \\ 3 & -1 & 1 & 1 & 0 & 2 & 0 & 1 \\ -1 & -3 & 1 & 0 & 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 1 & 3 & 1 & -1 & -2 \\ 1 & -1 & -3 & -2 & 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 & -2 & 0 & 1 & -3 \\ 0 & -2 & 0 & 1 & -1 & -1 & -3 & -1 \\ -1 & 0 & -2 & 3 & -1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 & -1 & 0 & 2 & -1 & 1 \\ 0 & -1 & 0 & 2 & 1 & -1 & 0 & 2 \\ 0 & -2 & 1 & 1 & 0 & 0 & 1 & -2 \\ -2 & 0 & -1 & 0 & 2 & 0 & -1 & -1 \\ -1 & 0 & 2 & 0 & -1 & -1 & -2 & 0 \\ -2 & 0 & 1 & -1 & 0 & 0 & 2 & 1 \\ -1 & 1 & 0 & 2 & -1 & 2 & 0 & 0 \\ -1 & -1 & -2 & 0 & -2 & -1 & 0 & 0 \end{bmatrix}$$

An Example (Cont.d)

Complexity Analysis

- Operation comparison with matrix multiplication

Operation	Fast simplified ICT	Matrix multiplication
Addition	150	240
Multiplication	0	256
Shifting	32	0

- Execution time comparison with matrix multiplication

  Transform 10,000 data blocks using 3.2GHz CPU and the data in the blocks are uniformly distributed in [-256,255]

	Fast algorithm	Matrix multiplication	Saving time
DCT	0.035 s	0.550 s	93.6%
Simplified ICT	0.025 s	0.293 s	91.4%
T_{8o}	0.004 s	0.036 s	88.9%

Conclusion and Future Work

- In this paper, a universal approach to developing fast algorithms for simplified order-16 ICT is proposed.
  - A general transform matrix for simplified order-16 ICT
  - Decomposed matrix multiplication to addition and shifting operations by a universal method

  Save 90% of the computational time compared with matrix multiplication
- Future work
  - When decomposing the odd part
    - Relax the constraints of each M_i matrices and explore whether number of operations can be reduced
    - Instead of exhaustive search, use new algorithm to search for a set of orthogonal vectors among a pool of vectors

Thank you!

Q&A